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Bayesian model updating of higher-dimensional dynamic systems

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1 SUMMARY

Model updating using measured system response, with or without excitation measured, has vast applications in structural health monitoring, response prediction, reliability and risk assessment, and structural control. There always exist modeling errors and uncertainties associated with the process of constructing a mathematical model of a structure and possibly its input loading. A fully probabilistic Bayesian model updating approach provides a robust and rigorous framework for these applications due to its ability to characterize modeling uncertainties associated with the underlying structural system and to its exclusive foundation on the probability axioms. The plausibility of each structural model within a class M of models is quantified by the updated joint probability density function $p(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M}$ (posterior PDF) of the uncertain model parameters $\boldsymbol{\theta} \in \mathbb{R}^D$, given the data D. By Bayes' theorem, the posterior PDF of $\boldsymbol{\theta}$ given the data $\mathcal{D}$ is given by: $p(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M}) = c^{-1}p(\mathcal{D}|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})$ where $c = p(D|\mathcal{M})$ is the normalizing constant which makes the probability volume under the posterior PDF equal to unity; $p(\mathcal{D}|\boldsymbol{\theta}, \mathcal{M})$ is the likelihood function based on the predictive PDF for the response given by model M; $p(\boldsymbol{\theta}|\mathcal{M})$ is the prior PDF in which one can incorporate engineering judgment through experience or previous analysis to quantify the initial plausibility of each model defined by the value of the parameters $\boldsymbol{\theta}$. Based on data D, a Bayesian model class M can be classified into 3 different cases (Beck & Katafygiotis 1998, Katafygiotis & Lam 2002): globally identifiable, locally identifiable and unidentifiable.

The Bayesian approach requires the evaluation of multi-dimensional integrals and this usually cannot be done analytically. Laplace's method of asymptotic approximation (Katafygiotis & Beck 1998) has been used in the past, which is based on a Gaussian approximation to the posterior PDF. However, the accuracy of such an approximation is questionable when (i) the amount of data is not sufficiently large or (ii) the chosen class of models turns out to be unidentifiable based on the available data. Also, such approximation requires a non-convex optimization in what is usually a high-dimensional parameter space, which is computationally challenging, especially when the model class is not globally identifiable.

Thus, in recent years, focus has shifted to using stochastic simulation methods in which samples consistent with the posterior PDF $p(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M})$ are generated. There are several difficulties related to this sampling: (i) the normalizing constant c in Bayes' theorem is unknown a priori and its evaluation requires a high-dimensional integration over the uncertain parameter space; (ii) the high probability content of $p(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M})$ occupies a much smaller volume than that of the prior PDF. Thus, samples in the high-probability density region of $p(\boldsymbol{\theta}|\mathcal{D}, \mathcal{M})$ cannot be generated by simulating from the prior PDF from which direct Monte Carlo simulation is usually possible. To tackle the aforementioned difficulties, Markov Chain Monte Carlo (MCMC) simulation methods (Beck & Au 2002, Ching et al. 2006, Ching & Cheng 2006) were proposed.

In this paper, we show how the Hybrid Monte Carlo method (HMCM) can be used to solve higher-dimensional Bayesian model updating problems. Improved variants of HMCM are proposed. The problem of convergence assessment is also addressed. Hybrid Monte Carlo Method (HMCM) is a MCMC technique for sampling from complex distributions by combining Gibbs sampling, Metropolis-Hasting (MH) algorithm acceptance rule and deterministic dynamical methods. By avoiding the random walk behavior of the MH algorithm through dynamical methods, HMCM can be much more efficient. The advantage of HMCM is even more pronounced when sampling highly-correlated parameters from posterior distributions which are often encountered in Bayesian structural model updating. Here the modified procedure suggested by Neal (1994) is also proposed in this paper to further improve HMCM.

HMCM procedure requires calculating the gradient of the negative natural log of the posterior PDF. For efficient calculation of gradient, we propose a way based on the idea of "algorithmic differentiation" (Rall, 1981; Kagiwada et al., 1986) which can be computationally more efficient than the finite difference by the number of times equal to the dimension of uncertain parameter vector $\boldsymbol{\theta}$.

A new method is proposed for monitoring and comparing the convergence of different simulation methods using the posterior samples and the gradient of negative natural log of posterior PDF.

Two examples are considered. The first example is a 30-dimensional posterior PDF with: (a) a Gaussian likelihood with very highly correlated parameters. The objective is to compare the performance of the following methods: (1) TMCMC (Ching & Cheng 2006) using local random walk in the Metropolis algorithm; (2) HMCM started by sampling from optimizing the posterior PDF (HM0); (3) as in (2) but using the improved acceptance probability procedure (HM1). All 3 methods use approximately the same number of computations. In this example, it can be seen that both HM0 and HM1 can perform better than TMCMC.

The second example is a 10-story linear shear building. The total acceleration on the ground, the first floor and the roof are measured. The objective is to estimate the mass m_i, damping coefficient c_i, and stiffness parameter k_i for each story. HM0 is applied to this problem. The sample mean and coefficient of variation (c.o.v) estimates of the structural parameters, along with the exact values of the parameters, are compared. It can be seen that the c.o.v. of the estimates are quite small and the estimate of the structural parameters is reasonably accurate even prediction and measurement errors in this problem are large.

The numerical examples show that the proposed methodology is feasible for solving model updating problems in higher dimensional parameter spaces.